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Mathematical description of protein extraction from muscle tissue of hydrobionts and determination of the effective molecular diffusion coefficient

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Abstract

The purpose of this work is to obtain an array of experimental data, kinetic dependences of the protein extracting from the particles of dispersed muscle tissue of hydrobionts (Alaska pollack), its mathematical description, and to find the molecular diffusion coefficients of proteins in the system of equations using solving the inverse problem of the modeling. This work's novelty is to study swelling and protein extraction from animal tissue and its mathematical description. This study has established that the diffusion of proteins from the muscle tissue of hydrobionts during stirring suspension (raw material-extractant) occurs at a significantly higher rate than diffusion from raw materials of plant origin. It indicates the contribution of a convective component to the swelling process and dissolution of particles due to intensive blending and easy differentiability of muscular tissue particles of hydrobionts. Based on the solution of the mathematical model, the molecular diffusion coefficient of proteins from raw materials of animal origin (muscle tissue of hydrobionts) $(3\div8)\times10^{-9}$ m^2/s, which exceeds the diffusion coefficients of carbohydrates from raw materials of plant origin (e.g., sugar 0.3×10^{-9} m^2/s) by more than an order of magnitude. Researchers can use the obtained molecular diffusion coefficient to calculate the parameters of the diffusion process of proteins during its extraction from dispersed particles of hydrobionts.

Keywords: mathematical description; mathematical modeling; molecular diffusion coefficient; protein extraction; waste recycling

Introduction

Among many typical processes of chemical technologies, protein extraction from raw materials of animal origin has attracted the least attention. In the food industry, complex technologies of raw material processing include processes of protein extraction from agricultural raw materials and wastes of its processing. In fish farming in aquaculture, there is a problem of disposal of fish processing waste, the solution of which allows to reduce the burden on the environment and obtain valuable proteins, lipids, and biologically active substances. The processes of nutrient extraction from raw materials of plant origin have been investigated in the most detail. Studies on the mathematical description of extracting sugar, vegetable oils (Tyulkova *et al.*, 2013), and biologically active compounds from raw materials were reported (Shishatskii *et al.*, 2015; Xie Y. *et al.*, 2020). The values of effective molecular diffusion coefficients, effective rate constants, and activation energy were determined in the above-mentioned processes (Nagai *et al.*, 2000; Jafari *et al.*, 2020).

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It is of paramount importance to define the value of the molecular diffusion coefficient as it allows us to calculate parameters for many technological processes and equipment in theory and practice.

Protein extraction from hydrobiont muscle tissue fundamentally differs from protein extraction from plant tissue. It is due to the presence of polysaccharides in the cell wall of plant tissue, which significantly slows down the extraction of proteins. Whereas in the structure of animal tissue of hydrobionts, collagen fibers are easily dissolved in the extractant, which facilitates the diffusion process. Swelling and breaking of chemical bonds were found to accompany the protein extraction process (Ciftci *et al.*, 2012). Protein-containing raw materials are known to have a cellular porous structure. Pore diffusion is the rate-limiting step when extracting different substances (Kafarov *et al.*, 2019). Diffusion of the extractant into the pores of the tissue to swelling of the raw material particles, which somewhat complicates the mathematical description of the extraction process. Therefore, the mass transfer process was determined by the swelling and presence of a chemical reaction between a proteinaceous material and extractant rather than by directly altering cell membrane permeability and destruction (Nagornov *et al.*, 2015).

There is a lack of data that enables the mathematically describing and modeling of protein extraction processes from animal tissues in the technologies of obtaining protein hydrolysates, isolates, and enzymes.

This work's novelty is to study the extraction of proteins from animal tissue, its mathematical description, solving the inverse problem of the modeling and determining the molecular diffusion coefficient of proteins from raw materials of animal origin. Researchers can use the obtained molecular diffusion coefficient to calculate the parameters of the diffusion process of proteins during its extraction from dispersed particles of hydrobionts.

Therefore, the aim of this study was to obtain an array of experimental data on extracting protein from the particles of dispersed muscle tissue of hydrobionts (Alaska pollack) to solve the inverse problem – description of unknown parameters in the system of equations describing the process under investigation, based on the obtained kinetic dependence of the extraction process.

The purpose of this work is to obtain an array of experimental data, kinetic dependences of the protein extracting from the particles of dispersed muscle tissue of hydrobionts (Alaska Pollack), its mathematical description and to find the molecular diffusion coefficients of proteins in the system of equations by means of solving the inverse problem of the modeling.

Materials and Methods

The muscle tissue of Alaska pollack was used as a model. It was dispersed by typical equipment up to the specific particle size: $(5.0\pm2)\times10^{-3}$ m. Then, the shredded tissue was placed in a water-salt medium with a salt concentration of 1% (to ensure electrical conductivity) to be processed in the cathode chamber of an electrolyzer. The mode was defined according to the parameters that provide the highest speed and maximum depth of dissolution.

The raw material was mixed with an electrolyte solution with specified water (ratio of 1:6 to ensure the fluidity of the suspension). Afterward, the mixture was processed through an electrolyzer, heated and thermostated up to the absolute visual structural breakdown. The insoluble components of the solution were bones and dermal tissue particles. The mathematical model of the process was constructed based on the kinetic curves of nutrients, obtained by us, extracted from fish particles into the extractant. The geometrical sizes of particles (R – spheres), their swelling degree, and their temperature are controlled during the whole process (Derkanosova *et al.*, 2011; Nagornov *et al.*, 2015; Perez *et al.*, 2011). Effect of mass changing presented in Figure 1A. Figure 1B presents changes in the mid-radius of tissue particles during the electrochemical process for extraction of nutrients from hydrobionts (according to data calculation and direct measurements). Figure 1C presents changing the temperature during the electrochemical extraction of nutrients from hydrobionts.

The solubility of protein particles (with different molecular weights and mass compositions) can be compared with the dissolution of a heterogeneous mixture of two or more chemical substances, which dissolved at different speeds, physically similar to leaching. A slowly dissolving component (protein of high molecular weight) constitutes a porous structure that determines the geometrical parameter of an R-const sphere through which the dissolvant and extraction products diffuse, which is similar to the process described in the paper (Kokotov *et al.*, 1970).

However, in this case, the inert matrix concerning the dissolvent is the low-molecular-weight proteins that are difficult to dissolve and form a spatial crosslinked structure. At the initial stage of the extraction process, high-molecular-weight proteins swell into the extractant and porous matrices that are constantly changing (Kao *et al.*, 2011). In this case, the insoluble components, such as bone and testaceous tissues, can be considered a factor in decreasing the active surface of the raw material.

Based on the kinetic curves and data on dry matter content in the extractant, the geometric parameters, and the

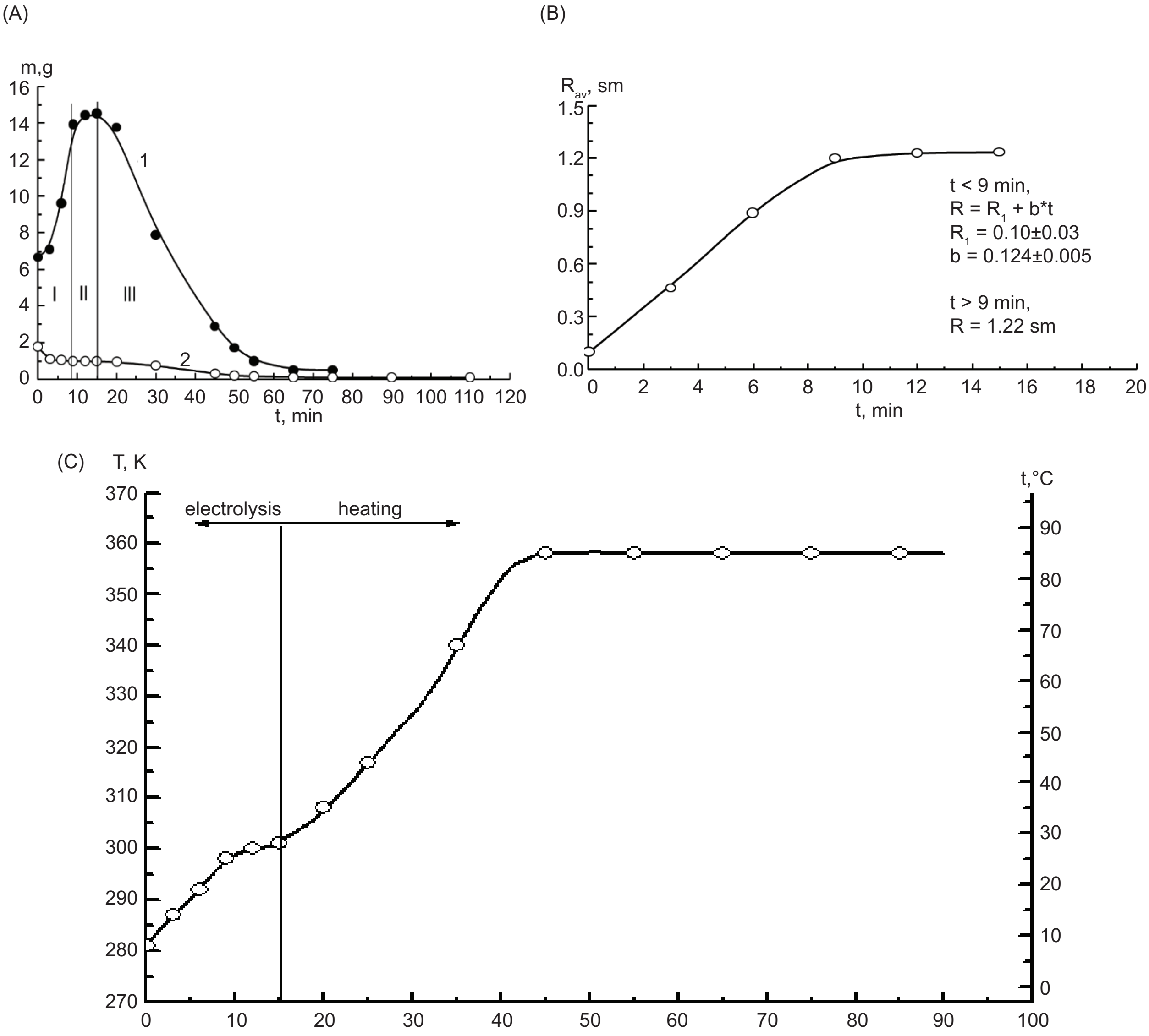

Figure 1. (A) Effect of mass changing in curves 1 (wet cake) and 2 – (dry cake) during the electrochemical extraction of nutrients from hydrobionts. I, II and III represents the approximate boundaries of the respective phases of the extraction process. (B) Changes in the mid-radius of tissue particles during the electrochemical process for extraction of nutrients from hydrobionts (according to data calculation and direct measurements). (C) Changing the temperature during the electrochemical extraction of nutrients from hydrobionts.

degree of swelling of the solid phase, the whole process of extraction–dissolution can be conditionally divided into two stages (Figure 2).

Stage I ($t = 0,15$ min): a relatively quick ongoing process of extraction, characterized by high speed, during which approximately 50% of the nutrients are extracted from the particles.

Figure 1A illustrates that the process of extraction is complicated by the simultaneous ongoing process of particle swellings, which finishes at the end of the stage, and the average radius of the particles increases to double. Swelling causes the extractant to penetrate the particles; its direction is opposite to the flux of the extractable matter from the particles. Thus, Stage I can be viewed as the process of extraction in a solid porous body–liquid system, which is complicated by swelling. The lack of influence of the hydrodynamic condition on the course of kinetic curves of stage I, proved experimentally, allows the limiting stage of mass transfer to be considered intra-diffusion.

The armor or bone should be considered as an inert impurity, which does not form integral spatial structures as its content was <10%. This effect can be reduced only

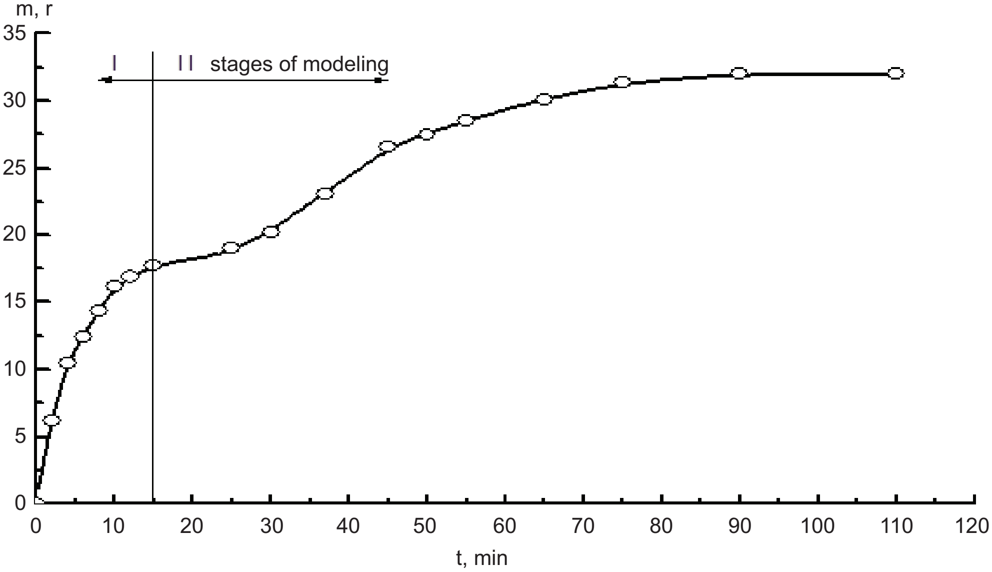

Figure 2. The content of dry substances in the solution during the extraction of nutrients from aquatic organisms using the electrochemical method.

to a specific decrease in the active surface area of the solid phase and can be considered as the introduction of the corresponding coefficient (Nagornov *et al.*, 2015; Bichkova, 2014).

The Stage II is characterized by the disintegration of the proteinaceous framework of high molecular weight proteins and the dissolution of muscular tissue.

Results

Mathematical modeling and solution of the inverse problem of the extraction process complicated by swelling

To describe the process, we must accept several assumptions allowing us to model an actual technological process.

Particles are spherical, identical in size, and isotropic by structure and composition, so the mass transfer problem in particles can be treated as one-dimensional and symmetric. Hence, pme, open parentence $C'(t)$ is a function only of coordinate R_0 (Nagornov *et al.*, 2015). The validity of these assumptions is based on the following facts:

1. Particles were obtained from tissue pushed out through the extruder's nozzle or spherical orifices. The size determinant is $R = R_0 = 3\times10^{-3}$ m at the initial moment;

   The size determinant R exceeds the size of the pores inside the particles. According to the data, the average diameter of the pores is 70×10^{-6} m (Ryckebosch *et al.*, 2012).

2. Mass transfer in step I: intra diffusion. The limiting step is the diffusion of substances inside the particles. It is possible to neglect mass transfer resistance both at the phase interface and inside the liquid phase.

According to Lewis-Whitman's theory (Romankov *et al.*, 1990), at the phase interface, diffusion can only occur by molecular mechanism. It happens due to the surface tension forces, as there are no convection currents in boundary layers. Depending on the nature of the materials, the hydrodynamic environment, and viscosity, the thickness of these layers (*d*) in the solid-liquid system is $0{,}1\text{-}30{,}0\times10^{-6}$ m. As the thickness of a boundary layer is small, the mass transfer resistance can be ignored (Romankov *et al.*, 1990; Protodyakonov *et al.*, 1987; Saravacos, 2014). The diffusion coefficient in the liquid phase is much higher than in solid systems. The diffusion coefficient in a liquid system is 10^{-9} m^2/s; in solids it is 10-13 m^2/s or less (Welty *et al.*, 2001; Serrano *et al.*, 2023; Hosseini *et al.*, 2023).

It happens since the extractant viscosity is very low, which is close to the viscosity of water (the ratio of raw material: extractant = 1:6). All particles are circulated with the extractant, and the convective component of diffusion exceeds a thousand times that of the molecular one $C_p=C_0$ (Romankov *et al.*, 1990).

3. The extraction and mass transfer of substances from solids occurs because of molecular diffusion. As

proven in (Gupalo *et al.*, 1971; Ghiaasiaan, 2018), the convective component of mass transfer in the particles is not considered.

4. Non-isothermal diffusion in the reaction mass caused by heating is negligible in comparison with the diffusion caused by a concentration gradient. Although Kafarov (Kafarov *et al.*, 2019) proposed a system of differential equations considering the imposition of the thermal diffusion component. He also showed that the thermal diffusion component in solutions was 10^3–10^5 times less than the concentration one. Thus, the overlay effects can be neglected.

The conditions of extraction in the system solid body–liquid Kafarov's criterion has the value of 10^{-2}-10^{-3}. It means that the concentration field is more inertial than the temperature one. The period of temperature nonstationarity in the solid particles (considering their small size and high thermal conductivity) is a negligible part of the period of extraction (Ostroushko *et al.*, 2012).

5. A chemical reaction does not complicate extraction.

6. Swelling of particles is caused by the absorption of the medium, which contains no protein component.

The porous spherical particle of muscle tissue has a mass flux inside that is caused by the molecular diffusion and existence of gradient concentrations of protein substances located in particles and surrounded by the solvent – a catholyte. Convective diffusion is absent as the catholyte at the initial time moment X=0 contains no substances of protein nature. Therefore, the value of diffusion stream sole- and water-soluble proteins from the particles of muscle tissue is calculated according to Fick's law:

$$j = -D_m \mathrm{grad} C \tag{1}$$

or in a differential form:

$$\mathrm{Partial}\frac{\partial C}{\partial t} = D_c \nabla^2 C \tag{2}$$

where ∇^2 is the Laplacian. D_m, D_c are the coefficients of mass - and concentration dependent.

However, there is Frolov and Romankov' (Romankov *et al.*, 1990) contradiction caused by different interpretations of the sense of a molecular diffusion coefficient D in these two equations. Equation (1) shows the coefficient of mass transfer and equation (2) shows the inertial properties of the concentration field, that is, the coefficient of concentration dependency.

The contradiction is eliminated, if instead of $C'(t)$ we consider $\mu'(t)$ the change of chemical potential. However, because of the practical convenience of using value C instead of m, we are limited by the equation (2) in spherical coordinates (r, q, φ) (Korn G., & Korn T., 1973):

$$\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial C}{\partial\theta}\right)+\frac{1}{\sin^2\theta\dfrac{\partial C}{\partial\theta^2}}\right] \tag{3}$$

In practice, instead of the molecular diffusion coefficient, the efficient diffusion coefficient is calculated considering the real mechanism of mass transfer in a capillary-porous body (Protodyakonov *et al.*, 1987). Considering the one-dimensional and symmetric problem, the concentration is treated as the function of only one spatial coordinate – r, and the equation becomes simpler:

$$\frac{\partial C}{\partial t} = D\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial C}{\partial r}\right)\right]$$

It is possible to show that porous spherical particles swell uniformly along the radial coordinate in the medium with a restricted volume. The following equations describe the change of the concentration of extracted component in a particle:

$$j_{\mathrm{d}} = D\frac{\mathrm{d}C}{\mathrm{d}r}$$

where j_d is the mass flow (the density of the diffusion flow of water- and salt-soluble proteins), which diffuses from the particle into the solution.

$$j_n = -c\varepsilon,$$

where j_n is the flow of the solvent that diffuses into particles, causing their swelling;

c is the concentration of the electrolyte containing no protein molecules;

$$\varepsilon = \frac{\mathrm{d}R_0}{\mathrm{d}t},$$

where R_0 is the radius of particles.

The total flow through the sphere with radius r:

$$j = D\frac{\partial C}{\partial \tau} - c\varepsilon$$

The common equation of the molecular diffusion in particles:

$$\begin{cases} \dfrac{dC}{dt} = divj = \dfrac{1}{\tau^2}\dfrac{\partial}{\partial r}(r^2 j) \\ \dfrac{\partial C}{\partial t} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(Dr^2\dfrac{\partial C}{\partial r} - \varepsilon r^2 C\right) \\ \dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial r^2} + \left(\dfrac{2D}{r} - \varepsilon\right)\dfrac{\partial C}{\partial r} - \dfrac{2}{r}\varepsilon C \end{cases} \tag{4}$$

The system of equations with initial and boundary conditions has the form:

$$\begin{cases} \dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial r^2} + \left(\dfrac{2D}{r} - \varepsilon\right)\dfrac{\partial C}{\partial r} - \dfrac{2}{r}\varepsilon C \\ V_\mathrm{p}\dfrac{\partial C_\mathrm{p}}{\partial t} = W\left[D\dfrac{\partial C}{\partial r} - \varepsilon C\right]_{\mathrm{r}=R_0} \\ C_{t=0} = C_0 \\ \varepsilon = \dfrac{\mathrm{d}R_0}{\mathrm{d}t} \\ \dfrac{\partial C}{\partial r}_{r=0} = 0 \\ C_\mathrm{p} = C_{r=R_0} \\ C_{\mathrm{p},t=0} = C_\mathrm{p}^0 \end{cases} \tag{5}$$

Where *C* is the concentration of the extracted component in the particles, the function of radial coordinate (*r*) and time (*t*);

> *D* is the efficient diffusion coefficient, the function of time;
>
> ε is the rate of change of linear dimensions of particles, the function of time;
>
> V_p is the volume of the solution, which is in contact with a solid phase, a function of time;
>
> *W* is the surface area of the particles, which are in contact with the solvent, function of time;
>
> *C* is the concentration of the extracted component in the liquid phase, a function of time;
>
> *R* is the radius of particles that are in contact with the solution, a function of time.

System (5) approximately describes the extraction of water - and salt-soluble components from hydrobiont particles in the experiments conducted. Since the average radius of the particles in Stage I changed in accordance with Figure 2, the corresponding expression for ε, according to the definition of this function, can be presented in the form:

$$\varepsilon = \begin{cases} b = const, & t < t_k \\ 0, & t > t_k \end{cases} \tag{6}$$

where *b* and t_k are constants that are equal for the system studied, 0, 124 cm/min and 9 min respectively. This means that at the end of Stage I, the kinetics of extraction of nutrients is described by the classical equation of internal diffusion in a limited volume:

$$\begin{cases} \dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial r^2} + \left(\dfrac{2D}{r}\right)\dfrac{\partial C}{\partial r} \\ V_p\dfrac{\partial C_p}{\partial t} = W\left[D\dfrac{\partial C}{\partial r}\right]_{r=R_0} \\ C_{t=t_k} = f(r) \\ \dfrac{\partial C}{\partial r}_{r=0} = 0 \\ C_p = C_{r=R_0} \\ C_{p,t=0} = C_p^0 \end{cases} \tag{7}$$

where f(*r*) is some distribution of the extracted component in time t_k.

Since the physical meaning of *C*(*r*,*t*) has to be continuous in the region of the change of arguments, then system (7) can be written in the form:

$$\begin{cases} \dfrac{\partial C}{\partial t} = D\dfrac{\partial^2 C}{\partial r^2} + \left(\dfrac{2D}{r}\right)\dfrac{\partial C}{\partial r} \\ V_p\dfrac{\partial C_p}{\partial t} = W\left[D\dfrac{\partial C}{\partial}\right]_{r=Rr_0} \\ C_{t=0} = f_0(r) \\ \dfrac{\partial C}{\partial r}_{r=0} = 0 \\ C_p = C_{r=R_0} \\ C_{p,t=0} = C_p^0 \end{cases} \tag{8}$$

where $\mathrm{f}_0(r)$ is some distribution of the extracted component at time 0, such that at time t_k the distribution of the extracted component in the particles corresponds to the function f(*r*). It should be noted that the solution of system (8) can reflect the distribution of a target component in particles from t_k untill the moment stage I finishes.

The description of the extraction kinetics of nutrients from hydrobionts at the first stage by means of systems (5)–(8) with the constant efficient diffusion coefficient is not correct as during the process the structure of the solid phase significantly changes. There are various target components in particles, and the extraction conditions are non-isothermal. The diffusion coefficient in systems (5)–(8) should be considered as functions of time. However, with $t>t_k$, swelling completed, the particle structure stabilized, and the low speed of a temperature change, the constant efficient diffusion coefficient characterizes this period and the approach represented works.

The solution of system (5) represents a complex computing challenge, which is hardly justified, considering that the first stage of extraction proceeds quickly enough and does not limit the processing speed in general. According to the available experimental data, knowing the properties of solutions of systems of equations of type (8), it is possible to estimate the efficient diffusion coefficient of nutrients in the swollen particles of hydrobionts.

As shown in studies by Derkanosova and Nagornov (Derkanosova *et al.*, 2011; Nagornov *et al.*, 2015), the systems of type (8) have the solution in the form of an infinite convergent power series of the form:

$$C\left(r,t\right)=C_{\infty}+\sum_{n=1}^{\infty}B_n\left(r,R,\mu_n\right)\exp(-\mu_n^2F_{\text{o}}) \tag{9}$$

where r, R are the current and defining radius of a particle;

C_{∞} is the residual concentration of the target component in particles;

B_n is the coefficient, which is function μ_n and B_i;

μ_n is the corresponding root of the characteristic equation, which is determined by the boundary conditions, and the case with restricted volume (the boundary conditions of the III sort) have the form (Kokotov *et al.*, 1970)

$$\begin{gathered} tg\mu_n=\frac{3\mu_n}{3+\alpha\mu_n^2} \\ \alpha=\frac{V_p}{V_S} \end{gathered} \tag{10}$$

where V_p и V_S are the volumes, respectively, for the solid and liquid phases, which are in contact;

$$F_{\text{o}}\text{ equals numerator, },F_{\text{o}}=\frac{Dt}{R^2} \tag{11}$$

where F_0 is the criterion of Fourier or homo-chronicity (dimensionless time);

R is the radius of the particle.

The solution of the diffusion equation of a target component from a sphere in the dimensionless variables expressed by a similar row (Bichkova, 2014; Kokotov *et al.*, 1970):

$$\begin{gathered} F(t)=1-\sum_{n=1}^{\infty}B_n\left(R,\mu_n\right)\exp\left(-\mu_n^2F_0\right) \\ F(t)=\frac{M(t)-M_0}{M_{\infty}-M_0} \end{gathered} \tag{12}$$

where M_0 is the initial quantity of a target component (an average in a particle);

$M(t)$ is the quantity of a target component (an average in a particle) in t instant;

M_{∞} is the quantity of a target component (an average in a particle) when reaching the equilibrium state.

The transition from $C(r,t)$ to $F(t)$ is rather convenient, as experimental data contain the total amount of extractable substances while the profile of concentrations in the body remains unknown. The resultant $F(t)$ function does not depend on coordinates.

Using dimensionless variables in the solutions of the diffusion equation allows bringing all the curves to one curve by changing a scale, describing diffusion in bodies of one form but of different sizes and with different diffusion properties. The resultant curve is common for all bodies of this spherical form (Kokotov *et al.*, 1970).

Known properties of the equations of the spherical form (9), (12) are such that the series converges quite quickly, and for computing the functions with acceptable accuracy, 3-5 members of the series are usually enough (Korn G. *et al.*, 1973). When the process is close to its completion, the convergence of the series (12) improves and at $F>0.7$. F is well described by the first member of a series (Kokotov *et al.*, 1970), that is:

$$F(t)\approx1-B_1\exp\left(-\mu_1^2F_0\right),\text{ when }F>0.7 \tag{13}$$

or after transforming and taking the logarithm:

$$\ln\left(1-F(t)\right)\approx\ln\left(B_1\right)-\mu_1^2\frac{D}{R^2}t,\text{ when }F_0>0.7 \tag{14}$$

Thus, from the experimental data and equation (14), it is possible to assess the efficient diffusion coefficient by characterizing the process of the internal diffusion of nutrients from hydrobionts to the first stage of extraction.

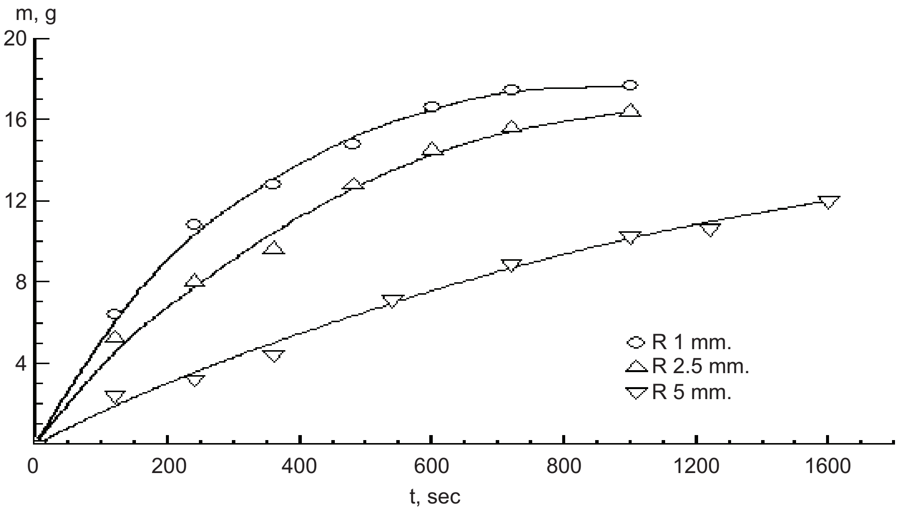

Figure 3. Content of the dry substances in the solution during the extraction of nutrients from hydrobionts by electrochemical method depending on time and radius of the original particles.

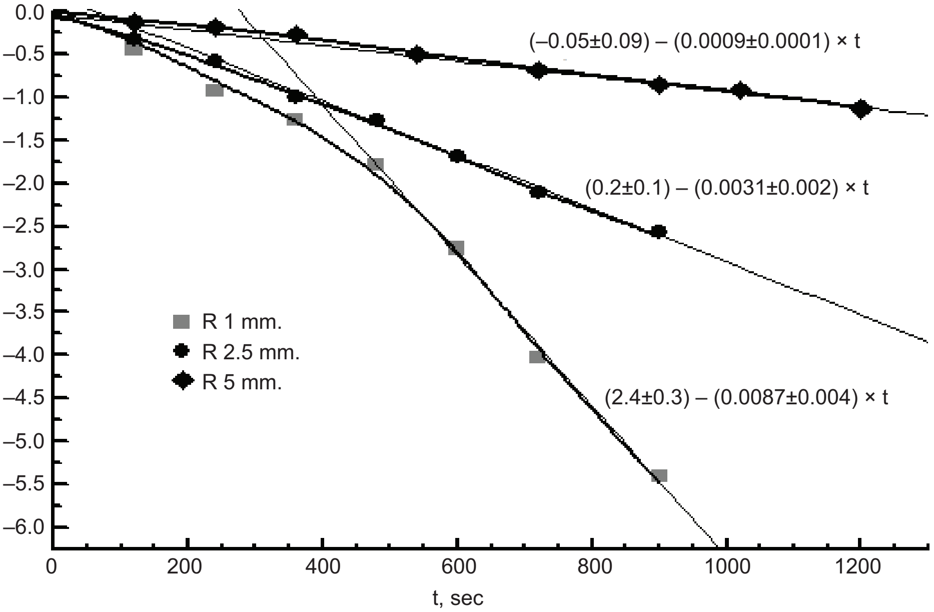

Figure 4. Results of experimental data according to equation (14).

Figure 3 represents the experimental data on the first stage of extraction of nutrients from spherical particles of hydrobionts of different initial sizes. Figure 4 shows the same data processed according to the equation (14). The maximum amount of solids that are capable of passing into a solution (the M analog) was considered identically for all the experiments conducted. The masses were taken for the extraction under identical conditions with the identical mass of the solid phase. The amount of solids was determined as an average value for all experiments – 17.2±0.2 g. The extraction was conducted with a prepared electrolyte with no nutrients in the solution at the beginning of the experiment. (C_0=0).

The angular coefficients of the lines (B_0), which are presented in Figure 5 for F>0.7, allow us to estimate the value of the effective diffusion coefficient:

$$D \approx B_0 \frac{R^2}{\mu_1^2} \tag{15}$$

Table 1. **Calculated values of coefficient B_0 and the effective diffusion coefficient.**

R_0, m	R, m (swelling)	B_0, 1/s	D, m²/s
1.0×10^{-3}	2.0×10^{-3}	8.7×10^{-3}	3×10^{-9}
2.5×10^{-3}	5.0×10^{-3}	3.1×10^{-3}	7×10^{-9}
5.0×10^{-3}	10.0×10^{-3}	0.9×10^{-3}	8×10^{-9}

As it is difficult to determine the average radius of the swollen particles (especially large) for calculations with a given formula, the radius accepted is twice larger than the initial one, which corresponds well with the data in Figure 2.

In the discussed experiments, the volume ratio of the phases (liquid and solid) is 3, corresponding with μ_1 – 3.4056. This value is obtained by solving the equation (10) with standard numerical methods (Vygodskiy, 2008).

The calculated values of the angular coefficients of the dependences (14) and the assessment of values of effective diffusion coefficients are given in Table 1. Despite the fact that the quantities in equation (15) have high precision, the effective diffusion coefficients should be considered as assessed in the order of their magnitude because the asymptotic approximation of the precise decision and other assumptions were used for the modeling.

Comparing the effective diffusion coefficients obtained for the particles of various radii, it is possible to consider that they coincide with the declared accuracy. It confirms the validity of using the intra-diffusion model.

Conclusions

As a result of this work the process of swelling of hydrobiont muscle tissue particles and extraction of proteins from them was investigated. This process is an integral stage of obtaining protein hydrolysates from raw materials of animal origin, used in the food industry.

Kinetic dependences of the process of protein extraction from particles of dispersed muscle tissue of hydrobionts were obtained, and its mathematical description and modeling were performed. The solution of the inverse modeling problem allowed us to obtain the molecular diffusion coefficient of proteins.

As a result of the mathematical description of the extraction process of muscle tissue particles, an important parameter from the theoretical and practical point of view, that is, the coefficient of a molecular diffusion $D = (3\div8)\times10^{-9}$ m²/s was defined. It has been established that this coefficient surpasses the coefficients of diffusion substances of biological nature from plant tissue by more than an order of magnitude (e.g., sugar 0.3×10^{-9} m²/c). It indicates the contribution of a convective component to the swelling process and dissolution of particles due to intensive blending and easy differentiability of muscular tissue particles of hydrobionts.

Researchers can use the obtained molecular diffusion coefficient to calculate the parameters of the diffusion process of proteins during its extraction from dispersed particles of hydrobionts.

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